Q1:

8 Marks

a) Reinforcement
b) Unsupervised
c) Reinforcement
d) Unsupervised
e) Reinforcement
f) Supervised
g) Unsupervised
h) Reinforcement.

1 mark each

Q2:

a) using median value approach.

Customer ID → Age
6 → 35

Age feature

Age:	20	22	25	30	35	35	40	45	50	60
Buy:	Yes	No	No	Yes	Yes	Yes	No	Yes	No	Yes

21 27 37 42 47 55

$\boxed{\text{Age}:21}$

6+ | 4−

<21 → 1+ | 0−

>21 → 5+ | 4−

Gini(Age<21)

$$= 1-\left(\frac{1}{1}\right)^2-\left(\frac{0}{1}\right)^2$$

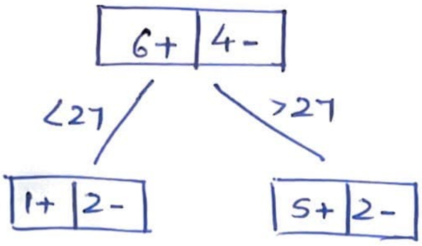
$$= 0$$

$$\text{Gini}(Age>21) = 1-\frac{25}{81}-\frac{16}{81}$$

$$= \frac{81-25-16}{81} = \frac{40}{81}$$

$$\text{Gini}(Age=21) = \frac{1}{10}\times 0 + \frac{\cancel{9}}{1\cancel{0}}\times\frac{4\cancel{0}}{\cancel{81}\,9} = \frac{4}{9} = 0.44$$

$\boxed{Age=27}$



$1-\left(\frac{1}{3}\right)^2-\left(\frac{2}{3}\right)^2$ $\qquad$ $1-\left(\frac{5}{7}\right)^2-\left(\frac{2}{7}\right)^2$

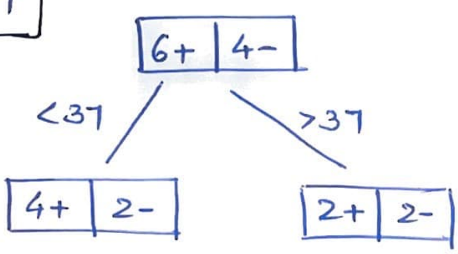
$\frac{9-1-4}{9} = \frac{4}{9}$ $\qquad$ $\frac{49-25-4}{49} = \frac{20}{49}$

$$Gini(Age=27) = \frac{3}{10} \times \frac{4}{9} + \frac{7}{10} \times \frac{20}{49}$$

$$= \frac{2}{15} + \frac{2}{7} = \frac{14+30}{105}$$

$$= 0.42$$

$\boxed{Age=37}$



$1-\left(\frac{4}{6}\right)^2-\left(\frac{2}{6}\right)^2$ $\qquad$ $1-\left(\frac{2}{4}\right)^2-\left(\frac{2}{4}\right)^2$

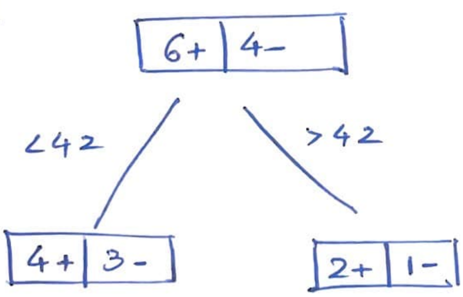
$\frac{36-16-4}{36} = \frac{16}{36} = \frac{4}{9}$ $\qquad$ $\frac{16-4-4}{16} = \frac{8}{16} = \frac{1}{2}$

$$Gini(Age=37) = \frac{6}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{1}{2}$$

$$= \frac{4}{15} + \frac{1}{5} = \frac{4+3}{15} = \frac{7}{15}$$

$$= 0.47$$

$\boxed{Age=42}$



$1-\left(\frac{4}{7}\right)^2-\left(\frac{3}{7}\right)^2$ $\qquad$ $1-\left(\frac{2}{3}\right)^2-\left(\frac{1}{3}\right)^2$

$\frac{49-16-9}{49} = \frac{24}{49}$ $\qquad$ $\frac{9-4-1}{9} = \frac{4}{9}$

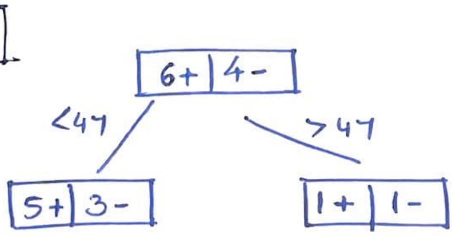
$$Gini(Age=42) = \frac{7}{10} \times \frac{24}{49} + \frac{3}{10} \times \frac{4}{9}$$

$$= \frac{12}{35} + \frac{2}{15}$$

$$= \frac{36+14}{105} = \frac{50}{105}$$

$$= 0.48$$

$\boxed{Age=47}$



$1-\left(\frac{5}{8}\right)^2-\left(\frac{3}{8}\right)^2$ $\qquad$ $1-\left(\frac{1}{2}\right)^2-\left(\frac{1}{2}\right)^2$

$\frac{64-25-9}{64} = \frac{30}{64}$ $\qquad$ $\frac{4-1-1}{4} = \frac{1}{2}$

$$Gini(Age=47) = \frac{8}{10} \times \frac{30}{64} + \frac{2}{10} \times \frac{1}{2}$$

$$= \frac{3}{8} + \frac{1}{10} = \frac{38}{80}$$

$$= 0.48$$

Age = 55

$6+ \mid 4-$

- <55: $5+ \mid 4-$
  $1-\left(\frac{5}{9}\right)^2-\left(\frac{4}{9}\right)^2$
  $\frac{81-25-16}{81} = \frac{40}{81}$
- >55: $1+ \mid 0-$
  $1-\left(\frac{1}{1}\right)^2$
  $=0$

$$\text{Gini (Age=55)} = \frac{9}{\cancel{10}} \times \frac{\cancel{40}}{81} + \frac{1}{10} \times 0$$
$$= \frac{4}{9} = 0.44$$

Income

$6+ \mid 4-$

- Low: $2+ \mid 2-$
  $1-\left(\frac{2}{4}\right)^2-\left(\frac{2}{4}\right)^2$
  $\frac{16-4-4}{16} = \frac{1}{2}$
- Medium: $1+ \mid 1-$
  $\frac{1}{2}$
- High: $3+ \mid 1-$
  $1-\frac{9}{16}-\frac{1}{16}$
  $\frac{16-10}{16} = \frac{6}{16} = \frac{3}{8}$

$$\text{Gini (Income)} = \frac{\cancel{4}^{\,2}}{\cancel{10}_{\,5}} \times \frac{1}{\cancel{2}} + \frac{\cancel{2}}{10} \times \frac{1}{\cancel{2}} + \frac{{}^{2}\cancel{4}}{{}_{5}\cancel{10}} \times \frac{3}{8}$$
$$= \frac{1}{5} + \frac{1}{10} + \frac{6}{40}$$
$$= \frac{8+4+6}{40} = \frac{18}{40} = 0.45$$

Car Owner

$6+ \mid 4-$

- Yes: $5+ \mid 0-$
  0
- No: $1+ \mid 4-$
  $1-\left(\frac{1}{5}\right)^2-\left(\frac{4}{5}\right)^2$
  $\frac{25-1-16}{25} = \frac{8}{25}$

$$\text{Gini (Car)} = \frac{\cancel{5}}{10} \times \frac{8}{\cancel{25}_{5}} = \frac{8}{50} = 0.16$$

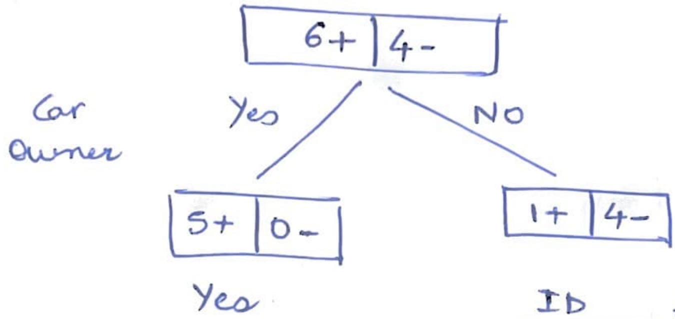
Credit Rating

$6+ \mid 4-$

- Low: $2+ \mid 1-$
  $1-\frac{4}{9}-\frac{1}{9}$
  $1-\frac{5}{9} = \frac{4}{9}$
- Medium: $2+ \mid 1-$
  $1-\left(\frac{2}{3}\right)^2-\left(\frac{1}{3}\right)^2$
  $=\frac{4}{9}$
- High: $2+ \mid 2-$
  $1-\frac{4}{16}-\frac{4}{16}$
  $=\frac{1}{2}$

$$\text{Gini (Credit Rating)} = \frac{3}{10} \times \frac{4}{9} + \frac{3}{10} \times \frac{4}{9} + \frac{4}{10} \times \frac{1}{2}$$
$$= \frac{2}{15} + \frac{2}{15} + \frac{1}{5} = \frac{7}{15}$$
$$= 0.467$$

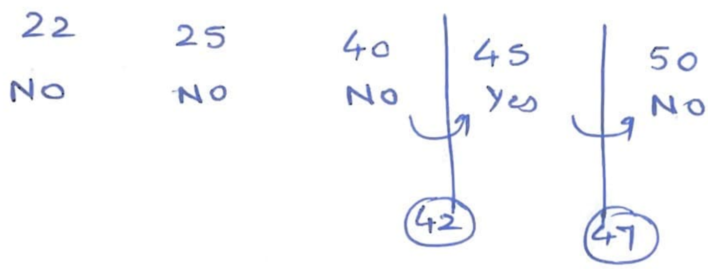
Car Owner feature is selected.



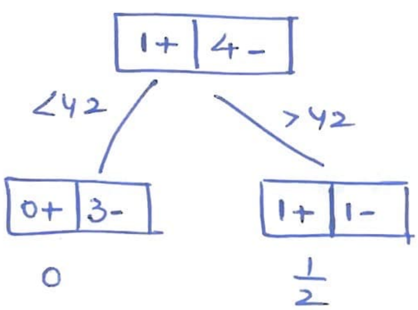
2nd level split

ID	Age	Income	CR	Buy
1	25	Low	H	NO
3	45	High	M	Yes
5	40	High	L	No
7	50	Medium	M	No
9	22	Low	H	NO

Age

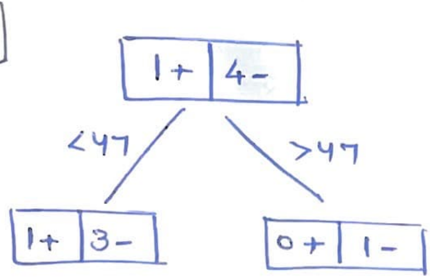


Age: 42



$Gini(Age=42) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5} = 0.2$

Age: 47



$1 - \frac{1}{16} - \frac{9}{16}$

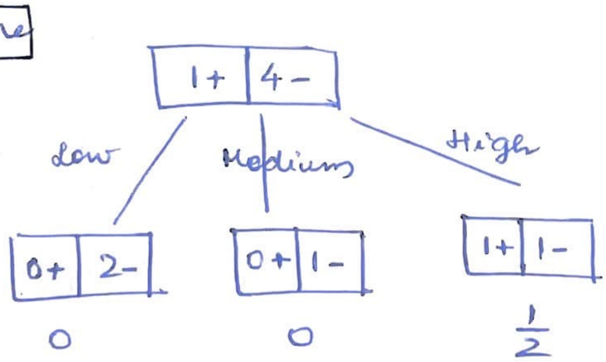
$= \frac{6}{16} = \frac{3}{8}$

$1 - \frac{0}{1} - \frac{1}{1}$

$= 0$

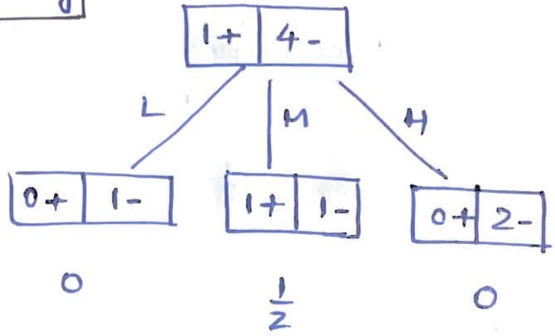
$Gini(Age=47) = \frac{4}{5} \times \frac{3}{8} = 0.3$

Income



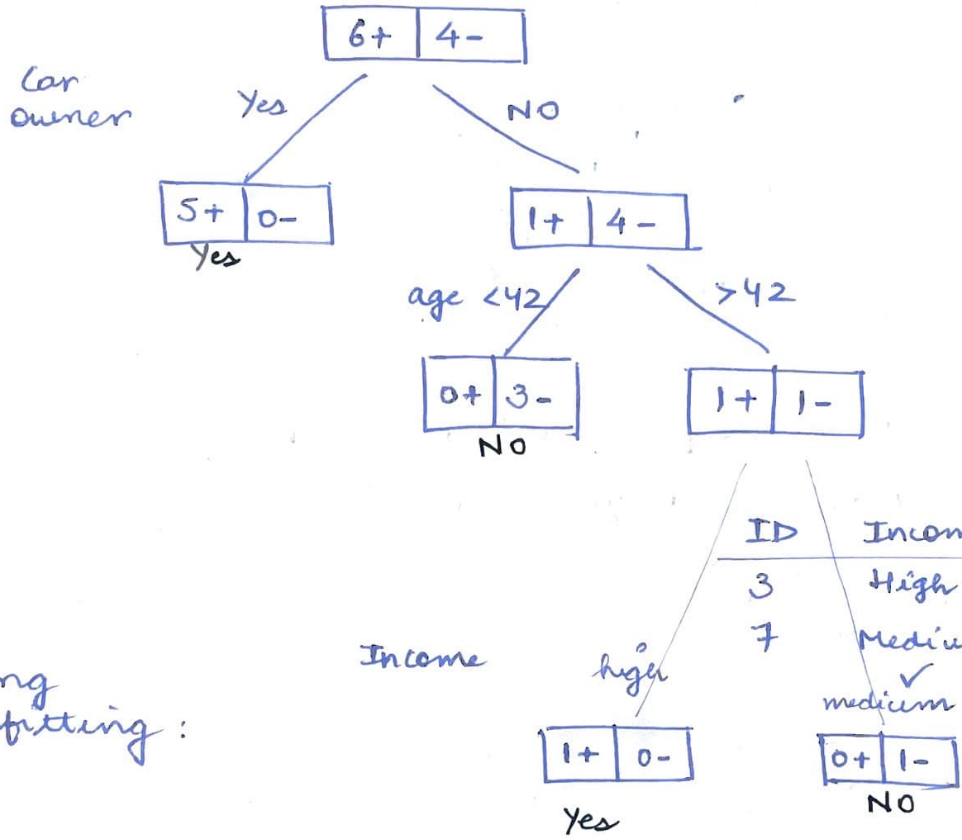
$Gini(Income) = \frac{2}{5} \times \frac{1}{2} = \frac{1}{5} = 0.2$

Credit Rating



$\text{Gini(CR)} = \frac{2}{5} \times \frac{1}{2} = 0.2$

Age: 42, CR and Income all give Gini as 0.2
But in CR and Income there are <2 samples. So we select age.



ID	Income	CR	Buy
3	High	M	Yes
7	Medium	M	NO

Q2 b)

Ignoring overfitting:

	actual	pred 2a	pred 2b
11	Y	N	N
12	N	N	N
13	Y	Y	Y
14	Y	Y	Y
15	N	Y	Y
16	N	N	N
17	N	Y	Y

$\text{Accuracy}_{2a} = \frac{\cancel{3}\,4}{7}$

$\text{Accuracy}_{2b} = \frac{\cancel{3}\,4}{7}$

Q3:

a) $P(y=D) = \frac{4}{10}$ $P(y=N) = \frac{6}{10}$

Age Group = O
Diet = NV
Exercise = R
Smoking habit = S

Age Group

	D	N
Y	2/4	3/6
O	2/4	3/6

Dietary Habit

	D	N
V	2/4	3/6
NV	2/4	3/6

Exercise

	D	N
I	3/4	2/6
R	1/4	4/6

Smoking

	D	N
S	3/4	2/6
NS	1/4	4/6

$P(y=D|x) = P(y=D) \cdot P(\text{age Group}=O|y=D) \cdot P(\text{diet}=NV|y=D) \cdot P(\text{exercise}=R|y=D) \cdot P(\text{smoking}=S|y=D)$

$= \frac{\cancel{4}}{10} \cdot \frac{\cancel{2}}{\cancel{4}} \cdot \frac{\cancel{2}}{\cancel{4}} \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{160} = 0.019$

$P(y=N|x) = \frac{\cancel{6}}{\cancel{10}_5} \cdot \frac{\cancel{3}}{\cancel{6}} \cdot \frac{\cancel{3}}{\cancel{6}_2} \cdot \frac{\cancel{4}^2}{\cancel{6}_2} \cdot \frac{\cancel{2}}{6} = \frac{1}{30} = 0.033$ ✓

b).

	C1 Distance (10,80)	C2 Distance (20,60)	Cluster.
(10,80)	0	22.36	C1
(11,79)	1.41	21.02	C1
(10,81)	1.0	23.26	C1
(20,60)	22.36	0	C2
(21,59)	23.71	1.41	C2
(20,61)	21.47	1	C2

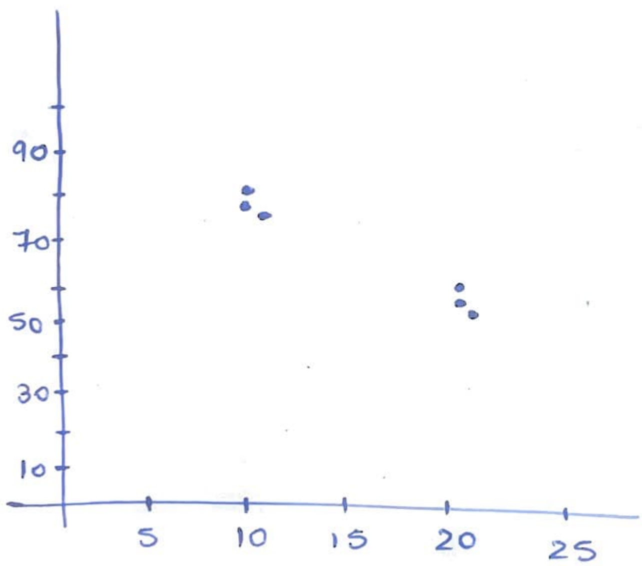
New centroids for $c_1' = (10.33, 80)$
New Centroids for $c_2' = (20.33, 60)$

	Distance from c_1'	Distance from c_2'	Cluster
(10,80)	0.33	22.51	c_1'
(11,79)	1.20	21.17	c_1'
(10,81)	1.05	23.40	c_1'
(20,60)	22.22	0.33	c_2'
(21,59)	23.56	1.20	c_2'
(20,61)	21.32	1.05	c_2'

$c_1'' = (10.33, 80)$

$c_2'' = (20.33, 60)$

$c_1' = c_1''$ and $c_2' = c_2''$ converged



Iteration 1: c1 intra cluster distance $= \frac{0+1.41+1.0}{3} = 0.803$

c2 intra cluster distance = 0.803

Iteration 2: c_1' intra cluster distance $= \frac{0.33+1.20+1.05}{3} = 0.86$

c_2' intra cluster distance = 0.86

c)

ID	Point	Distance from (27,2900)	Rank	
1	25,3000	100.02	2	low
2	30,2500	400.01	5	
3	28,2800	100.005	1	low
4	35,2000	900.04	6	
5	24,3200	300.015	4	low
6	40,1500	1400.06	7	
7	26,3100	200.00	3	High
8	45,1000	1900.04	8	

k=1 or 2 low risk

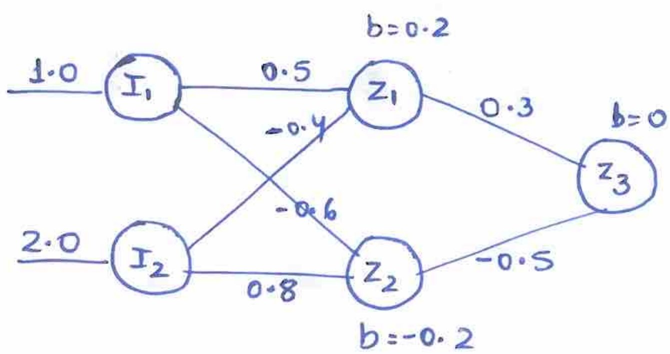
k=3 (includes outlier) low risk bcz of majority

k=4 low risk

outlier has no effect.

Q4:

a)



$Z_1 = 1.0 \times 0.5 + 2.0 \times -0.4 + 0.2 = -0.1$ $\quad a_1 = \text{Relu}(Z_1) = 0$

$Z_2 = 1.0 \times -0.6 + 2.0 \times 0.8 - 0.2 = 0.8$ $\quad a_2 = \text{Relu}(Z_2) = 0.8$

$Z_3 = 0 \times 0.3 + 0.8 \times -0.5 = -0.4$ $\quad a_3 = \text{Sigmoid}(-0.4)$

$$= \frac{1}{1+e^{0.4}} = 0.401$$

Binary cross entropy:

$$J = -\sum_{i=1}^{m} \left(y^{(i)} \log \hat{y}^{(i)} + (1-y^{(i)}) \log(1-\hat{y}^{(i)})\right)$$

$$= -\left(\log(1-0.401)\right) = -\log 0.599 = 0.2225$$

b) Backpropagation at output layer.

$$\frac{\partial L}{\partial w} = (\hat{y} - y) \cdot a$$

$$Z_1 Z_3 \text{ weight} = \underbrace{0.3}_{\text{old weight}} - \underbrace{(0.01)}_{\eta}\underbrace{(0.401)}_{(\hat{y}-y)} \cdot \underbrace{0}_{a} = 0.3$$

$$Z_2 Z_3 \text{ weight} = -0.5 - (0.01)(0.401)(0.8) = -0.5032$$

Hidden layer:

$$I_1 Z_1 \text{ weight} = 0.5 - \underbrace{(0.01)}_{\eta} \underbrace{(0.401)(0.3)}_{\text{Error}} \underbrace{(1.0)}_{\text{input}} = 0.4987$$

$$I_1 Z_2 \text{ weight} = -0.6 - (0.01)(0.401)(-0.5)(1.0) = -0.5979$$

$$I_2 Z_1 \text{ weight} = -0.4 - (0.01)(0.401)(0.3)(2.0) = -0.4024$$

$$I_2 Z_2 \text{ weight} = 0.8 - (0.01)(0.401)(-0.5)(2.0) = 0.8040$$

$$w_1 = \begin{bmatrix} 0.4987 & -0.5979 \\ -0.4024 & 0.8040 \end{bmatrix} \qquad w_2 = \begin{bmatrix} 0.3 \\ -.5032 \end{bmatrix}$$

Bias:

output layer:

$$b_0 = b_0 - \eta(\hat{y} - y) = 0 - (0.01)(0.401)$$
$$= -0.00401$$

Hidden layer:

$$b = 0.2 - (0.01)(0.401)(0.3) = 0.1987$$

$$b = -0.2 - (0.01)(0.401)(-0.5) = -0.1979$$

$$b_1 = \begin{bmatrix} 0.1987 \\ -0.1979 \end{bmatrix}$$

c).

$$\text{mean feature 1} = \frac{2.5 + 0.5 + 2.2}{3} = 1.733$$

$$\text{mean feature 2} = \frac{2.4 + 0.7 + 2.9}{3} = 2$$

mean centered data

$$\begin{bmatrix} 0.767 & -1.233 & 0.467 \\ 0.4 & -1.3 & 0.9 \end{bmatrix}$$

Covariance Matrix
$$\begin{bmatrix} cov(f1,f1) & cov(f1,f2) \\ cov(f2,f1) & cov(f2,f2) \end{bmatrix}$$

$$cov(f1,f1) = \frac{1}{2}\left[(0.767)^2 + (-1.233)^2 + (0.467)^2\right] = 0.0156$$

$$cov(f2,f2) = \frac{1}{2}\left[(0.4)^2 + (-1.3)^2 + (0.9)^2\right] = 0.1511$$

$$cov(f1,f2) = \frac{1}{2}\left[0.767 \times 0.4 + 1.233 \times 1.3 + 0.467 \times 0.9\right] = 0.0206$$

$$\Sigma = \begin{bmatrix} 0.0156 & 0.0206 \\ 0.0206 & 0.1511 \end{bmatrix}$$

$$|\Sigma - \lambda I| = 0 \qquad \begin{vmatrix} 0.0156-\lambda & 0.0206 \\ 0.0206 & 0.1511-\lambda \end{vmatrix} = 0$$

$$\lambda_1 = 0.1533 \qquad \lambda_2 = 0.0133$$

$$e_1 = \begin{bmatrix} -0.94 \\ 0.33 \end{bmatrix} \qquad e_2 = \begin{bmatrix} -0.33 \\ -0.44 \end{bmatrix}$$

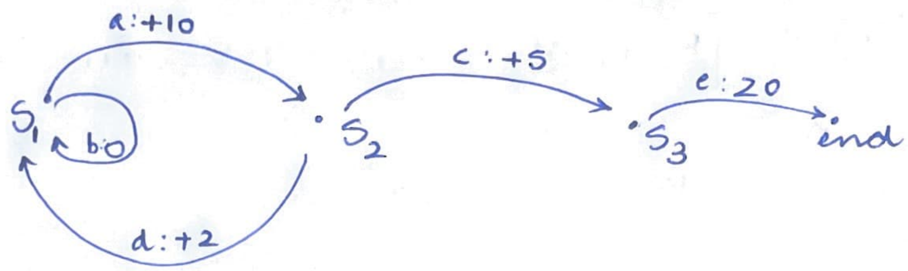
Projected Data = e_1^T Centered Data

$$= \begin{bmatrix} -0.94 & 0.33 \end{bmatrix} \begin{bmatrix} \quad \end{bmatrix}$$

$$= \begin{bmatrix} 0.8524 & -0.0350 & -0.0215 \end{bmatrix}$$

Q5:

a)



$\gamma = 0.5$ $\quad V_0(S_1) = 0$ $\quad V_0(S_2) = 0$ $\quad V_0(S_3) = 0$

$$V_{i+1}(s) = \max_a \sum_{s'} P(s'|s,a)\left[R(s,a,s') + \gamma V_i(s')\right]$$

$P(s'|s,a) = 1$ for $\forall a$

$V_1(S_1)$

action a:

$s' = S_2$ $\quad 10 + 0.5 \times 0 = 10$

action b:

$s' = S_1$ $\quad 0 + 0.5 \times 0 = 0$

$V_1(S_1) = \max(10, 0) = 10 \rightarrow$ action a.

$V_1(S_2)$

action c:

$s' = S_3$ $\quad 5 + 0.5 \times 0 = 5$

action d:

$s' = S_1$ $\quad 2 + 0.5 \times 0 = 2$

$V_1(S_2) = \max(5, 2) = 5 \rightarrow$ action c

$V_1(S_3)$

action e:

$s' =$ end $\quad 20 + 0.5 \times 0 = 20$

b)

$$V^{\pi}(s_t) = V^{\pi}(s_t) + \alpha\left[R(s_t, \pi(s), s_{t+1}) + \gamma V^{\pi}(s_{t+1}) - V^{\pi}(s_t)\right]$$

$\alpha = 0.1$ $\gamma = 1$ $R = -0.2$

$S_1 \rightarrow S_2$: $V(1) = 0 + 0.1\left(-0.2 + 1 \times 0 - 0\right) = \underline{-0.02}$

$S_2 \rightarrow S_3$: $V(2) = 0 + 0.1\left(-0.2 + 1 \times 0 - 0\right) = \underline{-0.02}$

$S_3, S_4 \ldots S_{15}$ $\}$ $V(S) = \underline{-0.02}$

$S_{15} \rightarrow G$: $V(15) = 0 + 0.1\left(5 + 1 \times 0 - 0\right) = \underline{0.5}$